Computational geometry and GIS for terrain modeling and simulation

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Abstract

In this paper, issues concerning GIS and computational geometry is first presented. The aim is to improve the understanding of the diversity of the possible interactions between computational geometry and GIS. Among issues addressed is the need to get a mesh representation which can enable to enhance the computation of surface flows in hydrologic and digital terrain models. Delaunay triangulation emerges to be an efficient tool because of its useful properties : realistic physical representation taking into account topological, geometrical and morphometric properties ; the flux across any edge with its neighbor is always normal to its boundary by definition and thus reduces the computational cost to evaluate flux across the volume boundary. Thus, we propose hydrological model for flood simulation based on Delaunay triangulation including processes such as overland flow and groundwater flow.
Keywords *: computational geometry, delaunay triangulation, digital terrain model, flood simulation.*

Résumé

Dans ce papier, une synthèse des problématiques concernant la géométrie algorithme et les systèmes d'information géographique (SIG) est d'abord effectuée. L'objectif est de cerner les possibles corrélations et interactions entre les SIG et la géométrie algorithmiques. Il en découle le besoin d'avoir une représentation qui facilite certains calculs des modèles hydrologiques et des modèles numériques de terrains. La triangulation de Delaunay s'est positionné comme un outil puissant à causes de ses propriétés particulières : représentation qui assure la conservations des propriétés topologiques, géométriques et morpho métriques ; coût d'évaluation des flux sur les bords réduits. Ainsi, nous proposons un modèle hydrologique pour la simulation d'inondation basé sur la triangulation de Delaunay et prenant en compte plusieurs processus dynamiques.
Mots clés *: géométrie algorithmique, triangulation de Delaunay, modèle numérique de terrain, modèle hydrologique.*

I. Introduction

GEOGRAPHICAL Information Science (GIS) is a multi-disciplinary area with contributions mainly coming from the spatial sciences (geodesy, geography and cartography), computer science (databases, artificial intelligence and computational geometry) and applied science (spatial planning, archeology, geology, civil engineering and biology). Thus, a substantial part of all activities performed by a GIS involves computing with the geometry of the data (such as location, shape, proximity, and spatial distribution) which calls for efficient methods to store, manipulate, analyze, and display such amounts of data (Van Oostrum, 1999). In the past, the importance of the data geometric aspects was often underestimated and the proposed solutions often suffered from lack of foundations, both from a geometric point of view (e.g the result of an operation defined just as the output of a certain algorithm) and from a computer science perspective (De Floriani et al, 2010). The needs for a solid theorical background and for high performance reasoning make the field of GIS an interesting source of problems to work on for computational geometers. Computational geometry is about the design and analysis of algorithms for geometric problems. The success of the field as research discipline can on the one hand be explained from the beauty of the problems studied and the solutions obtained, and on the other hand, by the many application domains (computer graphics, geographic information system, vision, path planning, image processing) in with geometric algorithms play a fundamental role (Berg et al, 2008).

This paper first reviews issues concerning GIS and computational geometry in order to improve the understanding of the diversity of the possible interactions between computational geometry and GIS and gives applications to terrain modeling and simulation. Indeed, real time hydrological processes simulation relies on spatial distribution representing discretization of representative terrain maps. Triangular network is typically the basis of such discretization. Thus some preprocessing become relevant, before running any simulation on the discretized space, to get a more realistic physical representation taking into account topological, geometrical and morphometric properties. Among other issues to be addressed is the need to get a mesh representation which can enable to enhance the computation of surface flows in hydrologic and digital terrain models. In this paper we propose a non-regular grid representation using computer geometric tools such as Voronoï diagram, Delaunay triangulation and geometric skeleton.

Section II discusses geographic information systems issues. Section III presents computational geometry tools that can be used in GIS. Section III introduces techniques and tools to generate digital terrain model based on irregular triangular networks. Section IV proposes a method of generating DTM based on Delaunay triangulation and java graphics 3d libraries. Section V presents a coupled hydrological and digital elevation model for flood simulation. Conceptual principles and schematic processing steps are given. Finally section VI presents conclusion and futures works.

II. GEOGRAPHY INFORMATION SYSTEM (GIS)

A geographic information system (GIS) can be seen as a system of hardware, software and procedures designed to support the capture, management, analysis, modeling and display of spatially referenced data for solving complex planning and management problems (De Floriani et al, 2010).

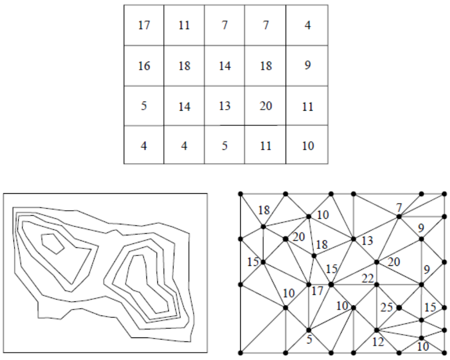

Figure 1. An elevation matrix, isoline model, and TIN for the same source data (Van Oostrum, 1995)

A. Data models and structures

There are basically three ways to store data : raster model, vector model and digital elevation model. The raster representation is a space oriented representation : the domain is subdivided in atomic regions similar to the pixels in digital image. Raster model provides direct information about a given location. However, besides their size raster structure has the disadvantage of providing an approximate geometry, whose accuracy is dependent on the resolution of the grid.

In vector model, points are stored with their coordinates according to some reference system, line segments by their two end points, and regions are defined by their bounding line segments. When incidence relations between points, segments, and regions are stored explicitly, we speak of a topological data structure. Raster model are more suitable to support queries by location while vector models are more suitable to support queries by content. Thus, hybrid representations, such as term digital elevation model (DEM), are more generally adopted. DEM includes the raster-based elevation matrix, and the vector-based isoline model and triangulated irregular network (Figure 1).The elevation matrix is a two-dimensional array of square cells, each cell storing a height value or elevation. The isoline model is a way to represent and visualize a real-valued function defined over the plane. The triangulated irregular network is a subdivision of the plane in non-uniform triangles.

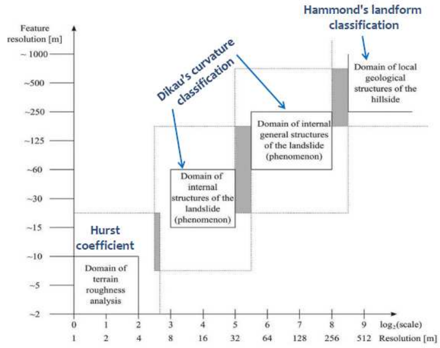

Figure 2. Topographical features classifications (Kalbermatten et al, 2011)

B. Terrain analysis

Topographic features are special points, lines and region that have a special meaning for describing the shape and the topologies of the terrain (Kalbermatten et al, 2011). They correspond to local differential properties of the terrain surface. In the case of TIN, point and linear features are vertices and chain of edges in the underlying triangulations ; area features are connected collection of triangles defining convex, concave and flat regions. Terrain analysis includes several geometric operations such as topological features extraction at resolution variable on the domain and computation of path on a surface (Figure 2). Rendering processes are based on realistic visualization and compression techniques. Defining a global reference system, terrain visibility and converting among different representation are challenges of primary interest in geographic data processing and require computation geometry tools.

III. COMPUTATIONAL GEOMETRY IN GIS

Computational geometry occurs in all stages of the GIS data cycle. Geometric algorithms are useful in data correction (after data acquisition and input), in data retrieval (through queries), data analysis (like map overlay and geostatistics), and data visualization (for maps and animations). For instance, line segment intersection by plane sweep solves map overlay, Voronoi diagrams are helpful in neighborhood analysis, Delaunay triangulations are widely used for terrain modeling, and geometric data structures help with efficient spatial indexing in large spatial data sets (Kreveld, 1999). Such problems are closely related to various well-known GIS operations like map overlay, buffer computation map generalization and spatio-temporal data mining and range searching.

A. Digital elevation model

Digital elevation (digital elevation, digital height, digital ground) modeling deals with generating a model for a terrain in a GIS that is realistic. The model is built on an initial data set which can consist of points with elevations or digitized contour lines with elevations. Generating a terrain that fits with the input data, the high-level information is a problem where several conflicting criteria have to be taken into account. The modeling problem and corresponding algorithms are a continuing direction of future research that has hardly been addressed from the computational geometry perspective (Kreveld, 1999). However, in the cases of TIN, Delaunay triangulation, which is optimal according the minimum angle, is more widely used.

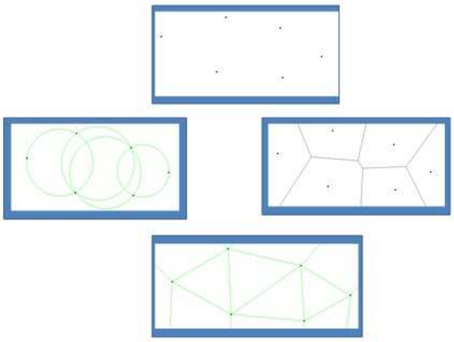

Figure 3. Set of points (top), Voronoï diagram (right), Delaunay circle (left), Delaunay triangulation of the point

B. *Delaunay triangulation*

From irregularly distributed data, either a regular grid or an irregular triangular network can be formed (Li et al, 2005 ; Ngom et al 2011). There are three basic requirements for TIN formation : for a given set of data points, the resulting TIN should be unique if the same algorithm is used ; the geometric shapes of resultant triangles are optimum (each triangle is nearly equilateral) ; each triangle is formed with nearest neighbor points, that is, the sum of the three edges of the triangle is minimum (Li et al 2005). In all the possible alternatives, Delaunay triangulation is the one most widely used because it satisfies all three requirements.

A Delaunay triangulation is a set of linked but non overlapping triangles uniquely defined if the points are in general position : no two points are identical, no three points lie on a common line, no four points lie in the same plane and no five points lie in the same sphere. The circumscribing circle of each triangle would not include any other points. As the Delaunay triangulation is a dual diagram of the Voronoi diagram (Figure 3), the Delaunay triangulation network can be formed either directly by algorithm or indirectly through the Voronoi diagram which is much easier in raster mode. The resulting triangulation network is independent of the starting point. Therefore, the selection of a starting point is only for the convenience of algorithm implementation. Delaunay's constraint on the mesh selects for nearly equilateral triangles, which evenly distribute the feature bias. TINs' vertices are derived from the features of the reference image, regardless of those features' positions. If we compare grid-based heightmaps to TINs at various resolutions, we would observe that the grid of the first heightmap grows and shrinks around a point of origin, while the TIN's mesh is anchored to the most important features of the reference image. Thus, TINs consistently represent multiple features of the terrain, while grid heightmaps can only consistently represent a feature if it lies at the grid's point of origin.

IV. COMPUTING THE GEOMETRIC TERRAIN MODEL

Modern acquisition techniques provide huge dataset that permit to represent terrain accurately. However, high representation accuracy is paid for in terms of high costs for storage and processing. So there must be a trading between the representation cost and accuracy. In this section we present a method of computing TIN.

A. *Computation of the Delaunay triangulation*

Delaunay triangulation can be formed in either dynamic or static mode. Static triangulation means that the triangulation network that has already been constructed will not be altered by adding new points in the formation process. In contrast, in dynamic triangulation, the network already constructed will be changed if a new point is added, so as to meet the Delaunay circumcircle principle.

A.1 Materials

We tested the triangulation with Java using a Window 7 environment. The program has been written in java but can be easily transferred in Java Applet so it can run in the internet. The real time rendering of the digital terrain model require Java 3D library. The Java3D library uses OpenGL or Direct3D operating software library to create the necessary algorithms to set up the 3D virtual universe.

A.2 Computation of the DTM based on TIN

The geometric terrain model is based on TIN computed from Delaunay triangulation algorithm.

- **Algorithms for computing the Delaunay triangulation**

  There are mainly four algorithms for computing the Delaunay triangulation : Incremental, Gift Wrap, divide and Conquer and QuickHull (Kreveld, 1996 ; Li et al, 2005). In this paper, the incremental algorithm and the Quickhull algorithm were used to compute respectively the 2D and 3D Delaunay triangulation. The incremental algorithm work as follow : to insert a new site, we walk across the triangulation, starting from the most recently created triangle, until we find the triangle that contains the new site. This triangle and any adjacent triangles that contain this new site in their circumcircle are eliminated and the resulting empty spot is re-triangulated. The expected time to insert a new site is roughly $O(n^{1/2})$ where n is the current number of sites (Chew, 2007). The *Quickhull* algorithm finds a point on the hull, then repeatedly looks for the next point until it returns to the start (Baber et al, 1996). The algorithm has *O(n log(n))* complexity. The data input is a text file containing points with x, y and z coordinates. The incremental algorithm is less time consuming but is difficult to generalize in 3D. That is why we chose the *QuickHull* algorithm for computing the 3D triangulation. However, there should be noted that by fixing the z coordinate, the *QuickHull* algorithm can also be used to compute 2D triangulation. The output of the *Delaunay triangulations* is a set of vertices and faces, printed in files that can be easily rendered.

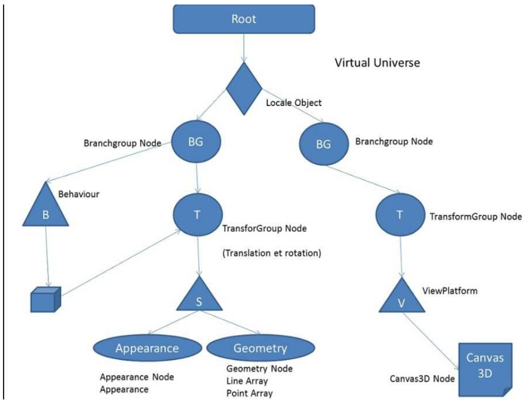

FIGURE 4. Scene describing the structure of the program

– *Implementation and visualization*
Even if many implementations of Delaunay triangulation have been proposed, to the best of our knowledge many of them suffer of platform dependencies and none is able to handle dynamic Delaunay triangulation as robust as it should be. In this paper, the implementation has been done in Java in order to have full control of the transformations on the model. To render 3D model using Java3D requires the construction of the Scene Graph to implement its 3D graphical operations. The Scene Graph of our application is divided in two main branches or Branchgroup. The diagram of Figure 4 outlines the position of each Java3d variable within the Scene Graph.

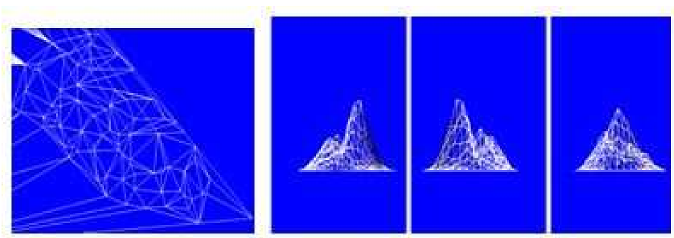

FIGURE 5. Triangulation of complex contour structure

The platform enables the visualization of the digital terrain as set of points or as a triangulated surface that can be associated with weight (Figure 4, Figure 5). As java3D has been used for the implementation, there is also possibility to look the surface from different point view through an interaction with the keyboard or the mouse. Besides, the program can be easily transferred in Java Applet application so it can be run on the internet.

V. FIRST STEPS TOWARDS FLOOD SIMULATION

Real time flood simulation relies on spatial distribution representing discretization of representative terrain maps. Among other issues to be addressed is the need to get a mesh representation which can enable to enhance the computation of surface flows in hydrologic and digital terrain models. In this paper we propose domain decomposition based on Delaunay triangulation. The model described here is based in the model presented by (Qu, 2004) resumed by Figure 6. The processes considered are overland flow, subsurface flow, evaporation and infiltration.

A. *Domain decomposition*

Delaynay triangulation is a realistic physical representation taking into account topological, geometrical and morphometric properties. Besides, the flux across any edge with its neighbor is always normal to its boundary by definition and thus reduces the computational cost to evaluate flux across the volume boundary. In the model, Delaunay triangulation is used to represent the watershed terrain and incorporate watershed boundaries, the stream network, soil type, geology, elevation contours or other feature particular to the domain. Starting points for the watershed domain start from a set of user defined control points : hydrographic points and critical terrain points (maximum or minimum of local surface, convexity or concavity ...).

B. *Gouverning Hydorlogical processes equations*

The governing equations are local system of ODE's representing multiple hydrological processes within triangle or element i including

– **Surface overland flow :**

$$\delta h/\delta t = P_0 - I - E_0 - Q_{oc} + \Sigma_{j=1}^{3} Q_s^U \tag{1}$$

Where h is the local water depth, Q_s^U is surface flow from element i to its neighbor j, P_o, I and E_0 are precipitation, infiltration and evaporation respectively.

– **Subsurface flow :**

$$\begin{cases} d\xi/dt = I - q^0 - ET_s \\ d\zeta/dt = q^0 + \Sigma_{j=1}^{3} Q_g^U - Q_l + Q_{gc} \end{cases} \tag{2}$$

where ξ is unsatured moisture storage of element I, ζ is satured moisture storage, q^0 is unsatured zone and satured zone ; I and ET are incoming infiltration and outgoing evapotranspiration at land surface respectively ; Q_s^U is lateral groundwater flow from element I to its neighbor Q_l is vertical leakage through and underlying confining bed ; Q_{gc} is discharge/recharge from/to aquifer to/from channel.

One advantage of this study is the flexibility in this constitution relationship and there is no intrinsic limitation to other possible forms or hydrological processes.

C. *Towards a model of flood simulation*

The processing steps of the model are resumed by Figure 6.

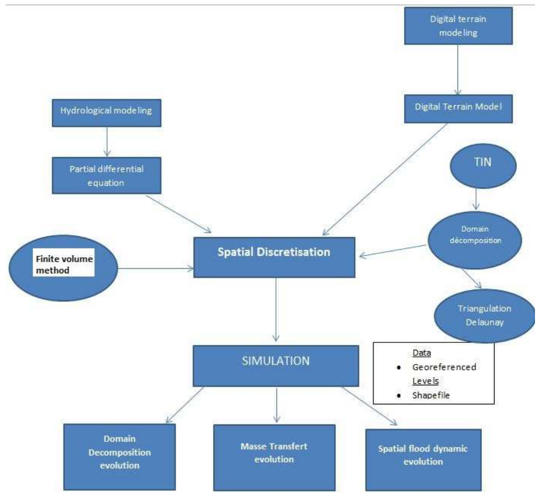

FIGURE 6. Step processing of the hydrological model based on DT domain decomposition

Spatial discretization of the hydrological model coupled with a digital elevation model of surface is first performed. The governing equations of hydrological model are based on a system of ODE (Equation 1, Equation 2) extracted from Saint Venant equations. The hydrological model will be integrated using a semi-discrete finite volume method. The finite volume elements are prisms projected from a Delaunay Triangulation based TIN. The model is designed to simulate hydrological processes such as water movement, water quality and sediment transport in surface and subsurface.

VI. CONCLUSION

In this paper we have first reviewed issues concerning computational geometry and GIS. At the state of the art, computational geometry offers rigorous and powerful tools for solving a variety of geometric problems in GIS. However, a number of problems that could be solved with computational geometry tools remain to be solved. We used java application based on the convex hull algorithm to build triangular irregular network for 2D set of points and 3D set of points. Java 3D graphics package was used to render the three dimensional model of TIN. In the last section, we introduce conceptual principles and steps processing for a hydrological model based on Delaunay triangulation terrain decomposition.

Futures works will focus on improving the digital terrain model by providing tools for the user to move through the virtual world, to supply elevation datasets, to drape images over the model, to generate statistics and perform simulation of hydrological processes over the model. Our aim is to create an interactive, real-time three-dimensional terrain model that can be rendered with free 3d graphics libraries, which may be used as a base for digital terrain analysis, natural phenomena real time simulation for decision making.

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